# PART 2

# Fluid Dynamics

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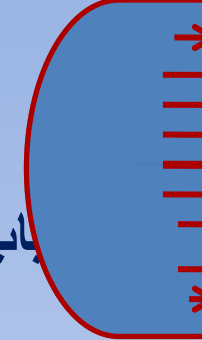
# General Concepts of fluids flow



***Fluid flow can be***

(Laminar or flowSteady ) ياب طبقى

(Turbulence flow or non steady) إنسياب مضطرب – عشوائى

***In steady (laminar) or non-steady.*** The *velocity v of the fluid particlesat any point remains constant* in time.

****Above a certain critical speed, fluid flow becomes turbulent; turbulent flow is irregular flow.**

**The term viscosity is commonly used in the description of fluid flow to characterize the degree of internal friction in the fluid.**
**This internal friction, or *viscous force,* is associated with the resistance that two adjacent layers of fluid have to moving relative**
**to each other. Viscosity causes part of the kinetic energy of a fluid to be converted to internal energy. This mechanism is similar to the one by which an object sliding on a rough horizontal surface loses kinetic energy.**

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**Because the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our model of ideal fluid flow, we make the following four assumptions**:

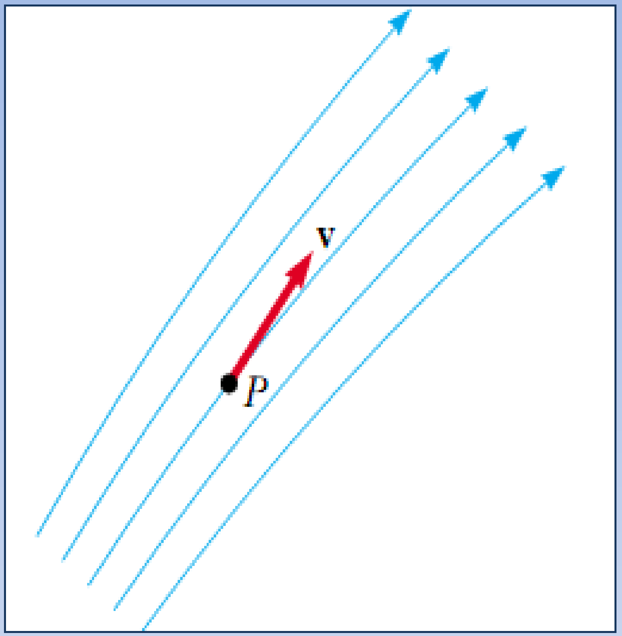
1. **The fluid is nonviscous**. In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force

2. **The flow is steady**. In steady (laminar) flow, the velocity of the fluid at each point remains constant.

3. **The fluid is incompressible**. The density of an incompressible fluid is constant.

4. **The flow is irrotational**. In irrotational flow, the fluid has no angular momentum about any point.

The path taken by a fluid particle under steady flow is called a streamline. The velocity of the particle is always tangent to the streamline, as shown in Figure. A set of streamlines like the ones shown in Figure form a *tube of flow. Note that fluid* particles cannot flow into or out of the sides of this tube; if they could, then the streamlines would cross each other.



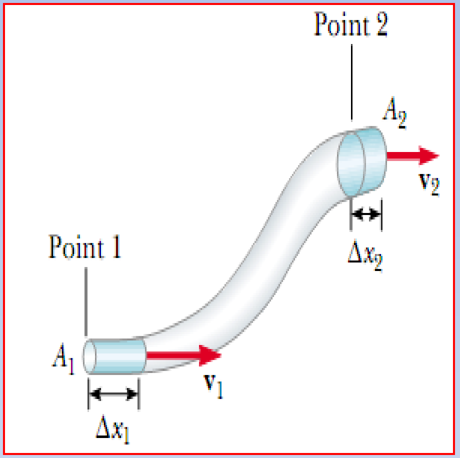

A particle in laminar flow follows a streamline, and at each point along its path the particle's velocity is tangent To the streamline

# The Continuity Equation

- *Let the speed be $v_1$ and $v_2$ for fluid particles at P and Q respectively.*
- *Let $A_1$ and $A_2$ are cross- sectional areas of the tube at the points P and Q,*
  *In the time $\Delta t$, fluid element travels approximately the distance $\Delta x_1 = v_1 \Delta t$*

*The mass of fluid $\Delta m_1$ crossing $A_1$ in the time interval $\Delta t$ is:*

$$\Delta m_1 = = \rho_1 A_1 \Delta x_1 = \rho_1 A_1 v_1 \Delta t$$




- *. The mass of fluid $\Delta m_2$ crossing $A_2$ in the same time interval $\Delta t$ is:*
  - $\Delta m_2 = \rho_2 A_2 v_2 \Delta t$
- *The fluid is incompressible, $\rho_1 = \rho_2 = \rho$ and because the flow is steady, then,*

$$\Delta m_1 = \Delta m_2 \quad \Rightarrow \quad \rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$$

- *Then*
- $A_1 v_1 = A_2 v_2 = \text{constant}$
- *Or* $A v = \text{constant}$
- *This expression is called the equation of continuity for fluids.*

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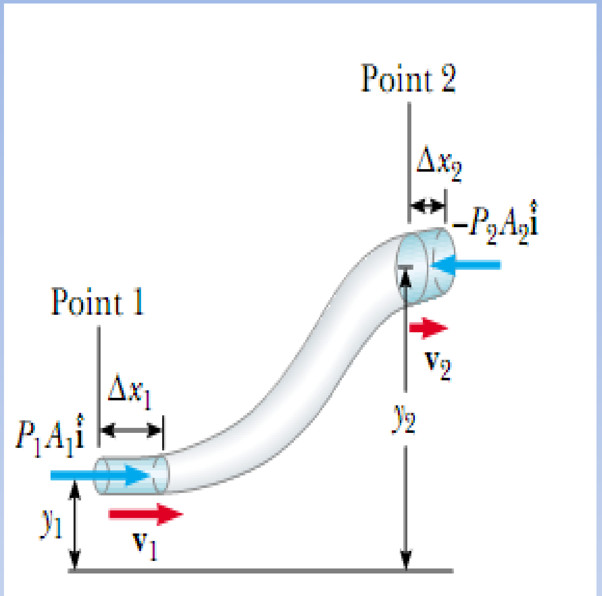
the product of the area and the fluid speed at all points along a pipe is constant for an incompressible fluid.

Example: **Each second, 5. 525 $m^3$ of water flows over the 670-m-wide cliff جرف of the Horseshoe Falls portion of Niagara Falls. The water is approximately 2 m deep as it reaches the cliff. What is its speed at that instant?**

# BERNOULLI'S EQUATION

Consider the flow of a segment of an ideal fluid through a nonuniform pipe in a time interval $\Delta t$, *as illustrated in Figure. At the beginning of the time interval,* the Segment of fluid consists of the blue shaded portion (portion 1) at the left and the unshaded portion. During the time interval, the left end of the segment moves to the right by a distance $\Delta x_1$ , which is the length of the blue shaded portion at the left. Meanwhile, the right end of the segment moves to the right through a distance $\Delta x_2$, which is the length of the blue shaded portion (portion 2) at the upper right of Figure.

Thus, at the end of the time interval, the segment of fluid consists of the unshaded portion and the blue shaded portion at the upper right.




A fluid in laminar flow through a constricted pipe. The volume of the shaded portion on the left is equal to the volume of the shaded portion on the right.

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**Now consider forces exerted on this segment by fluid to the left and the right of the segment. The force exerted by the fluid on the left end has a magnitude *P1A1***

**The work done by this force on the segment in a time interval $\Delta t$ *is* $W_1$**

$$W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1 = P_1 V$$

**. In a similar manner, the work done by the fluid to the right of the segment in the same time interval $\Delta t$ *is W***

$$W_2 = -P_2 A_2 \Delta x_2 = -P_2 V.$$

**This work is negative because the force on the segment of fluid is to the left and the displacement is to the right.**
**Thus, the net work done on the segment by these forces in the time interval $\Delta t$ *is***

$$W = (P_1 - P_2) V$$

**The volume of the shaded section on the left = the volume of the shaded section on the right**

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If *m is the mass that enters one* end and leaves the other in a time *t, then the change in the kinetic energy of this* mass is

$$\Delta K = \frac{1}{2} m v_2{}^2 - \frac{1}{2} m v_1{}^2$$

The change in gravitational potential energy is

$$\Delta U = mgy_2 - mgy_1$$

$$W = \Delta K + \Delta U \quad \longleftrightarrow \quad W = (P_1 - P_2)V$$

$$(P_1 - P_2)V = \frac{1}{2} m v_2{}^2 - \frac{1}{2} m v_1{}^2 + mgy_2 - mgy_1$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2{}^2 - \frac{1}{2}\rho v_1{}^2 + \rho g y_2 - \rho g y_1$$

$$P_1 + \frac{1}{2}\rho v_1{}^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2{}^2 + \rho g y_2$$

**Bernoulli's equation** as applied to an ideal fluid.

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$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

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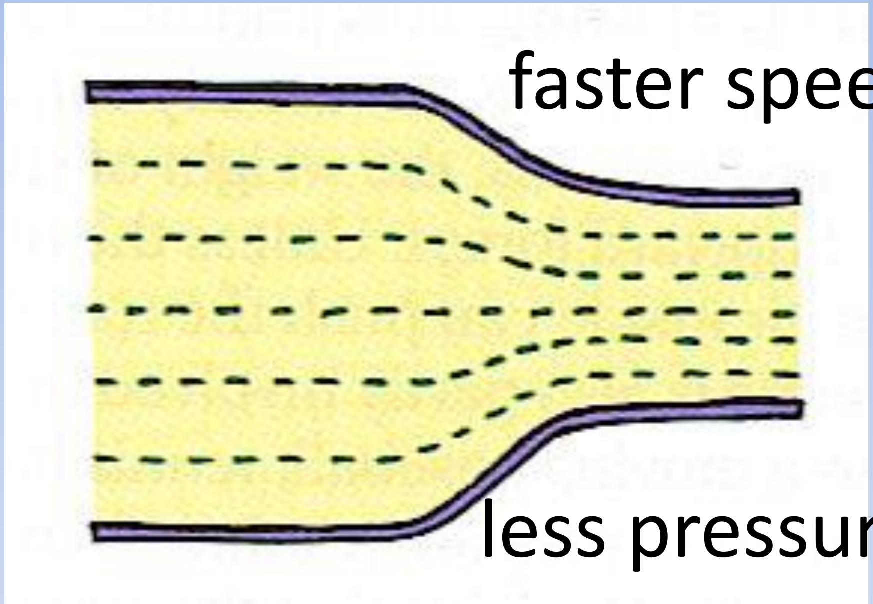
If no change in height:

$$P + \tfrac{1}{2}\rho v^2 = \text{constant}$$

slower speed

more pressure




As the speed of a fluid increases, the pressure in the fluid decreases.

$P \ \alpha \ 1/v$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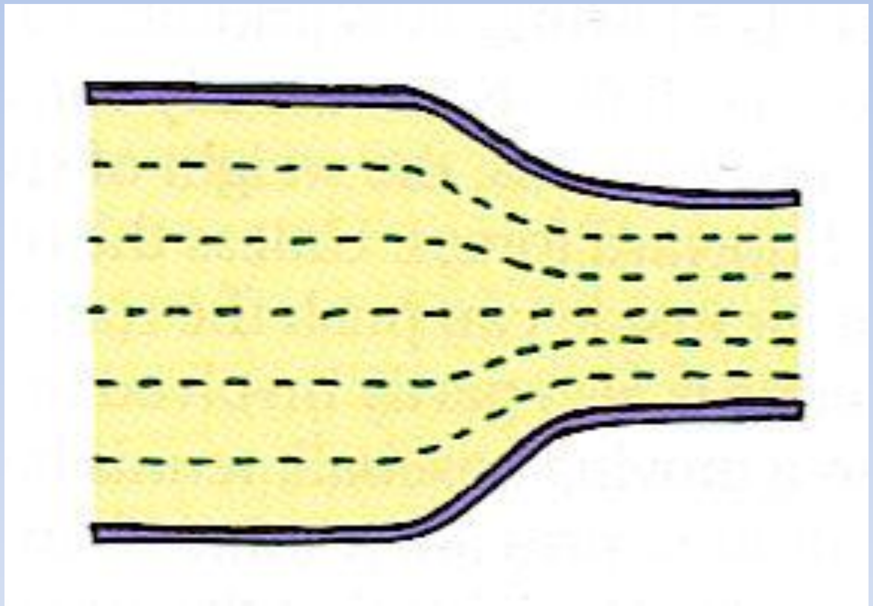
Motion of fluid → kinetic energy

Pressure in fluid → potential energy



KE + PE is constant

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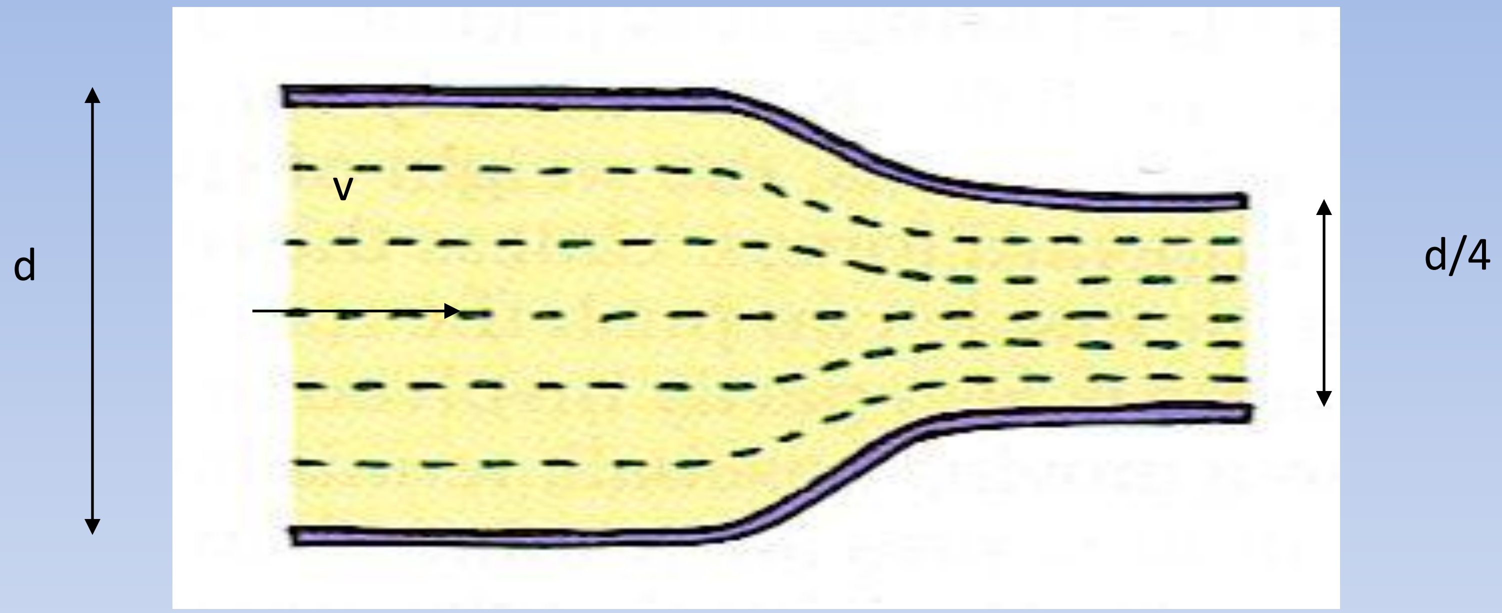


If incompressible fluid (density = ρ), change in pressure?

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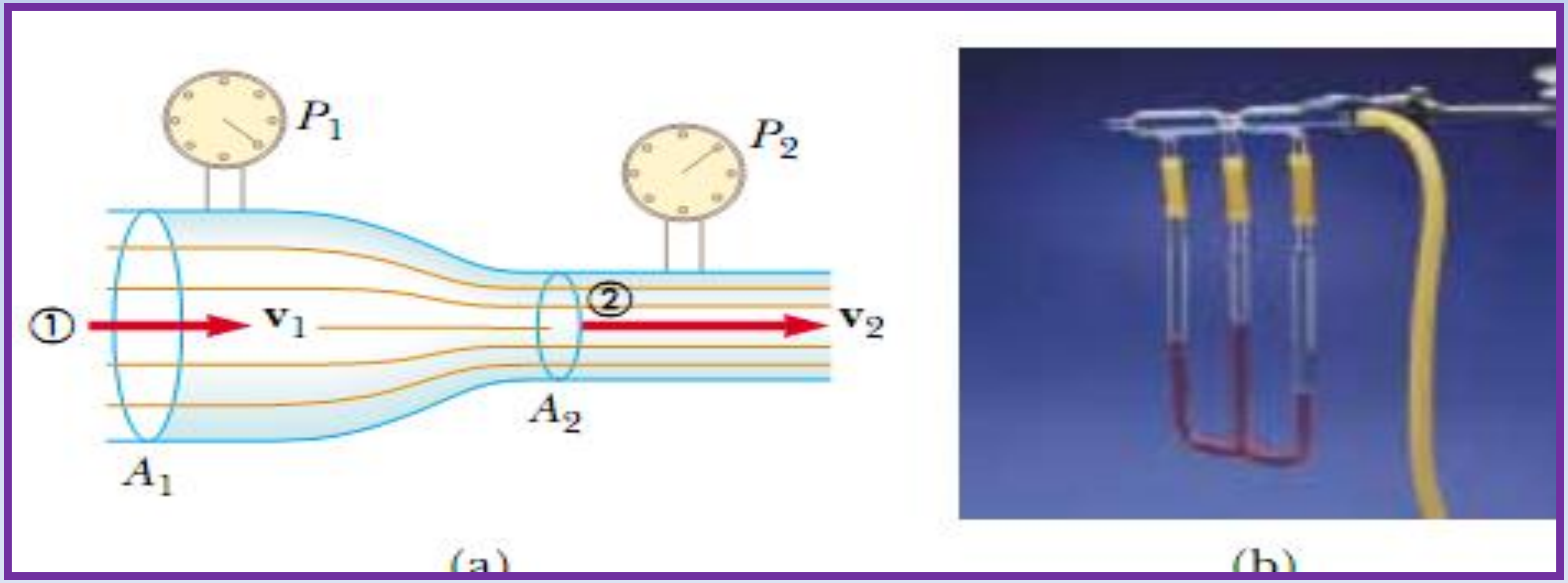
# Applications of Bernoulli's Principale

## The Venturi Tube

The horizontal constricted pipe illustrated in Figure known as a *Venturi tube,*

***Venturi tube**, can be used to measure the flow* **speed of an incompressible fluid. Determine the flow speed at point 2 if the pressure difference *P1-P2 is known*!**

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**Solution** Because the pipe is horizontal, $y_1 = y_2$, and applying Equation 14.8 to points 1 and 2 gives

$$(1) \qquad P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

From the equation of continuity, $A_1 v_1 = A_2 v_2$, we find that

$$(2) \qquad v_1 = \frac{A_2}{A_1} v_2$$

Substituting this expression into Equation (1) gives

$$P_1 + \tfrac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$v_2 = \boxed{A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}}$$

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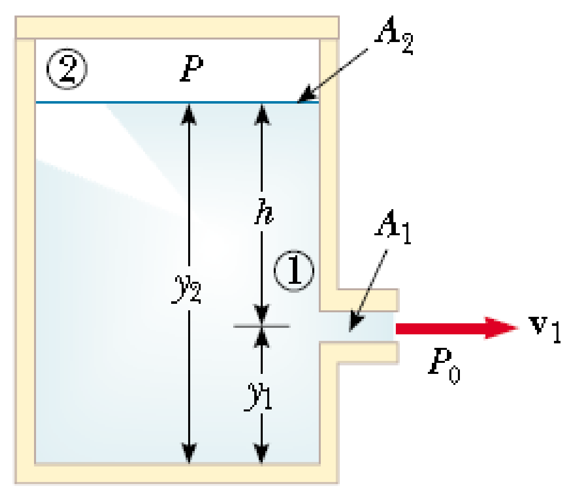
# Torricelli's Law

An enclosed tank containing a liquid of density $\rho$ has a hole in its side at a distance $y_1$ from the tank's bottom (Fig. 14.21). The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air above the liquid is maintained at a pressure $P$. Determine the speed of the liquid as it leaves the hole when the liquid's level is a distance $h$ above the hole.




**Solution** Because $A_2 \gg A_1$, the liquid is approximately at rest at the top of the tank, where the pressure is $P$. Applying Bernoulli's equation to points 1 and 2 and noting that at the hole $P_1$ is equal to atmospheric pressure $P_0$, we find that

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_0 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

But $y_2 - y_1 = h$; thus, this expression reduces to

$$v_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

**If the tank is open to the atmosphere, then** $P = P_0$ and $v_1 = \sqrt{2gh}$

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# SUMMARY

The **pressure** $P$ in a fluid is the force per unit area exerted by the fluid on a surface:

$$P \equiv \frac{F}{A} \tag{14.1}$$

In the SI system, pressure has units of newtons per square meter ($N/m^2$), and $1\ N/m^2 =$ 1 **pascal** (Pa).

The pressure in a fluid at rest varies with depth $h$ in the fluid according to the expression

$$P = P_0 + \rho g h \tag{14.4}$$

where $P_0$ is the pressure at $h = 0$ and $\rho$ is the density of the fluid, assumed uniform.

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**Pascal's law** states that when pressure is applied to an enclosed fluid, the pressure is transmitted undiminished to every point in the fluid and to every point on the walls of the container.

When an object is partially or fully submerged in a fluid, the fluid exerts on the object an upward force called the **buoyant force.** According to **Archimedes's principle,** the magnitude of the buoyant force is equal to the weight of the fluid displaced by the object:

$$B = \rho_{\text{fluid}}\, gV \tag{14.5}$$

You can understand various aspects of a fluid's dynamics by assuming that the fluid is nonviscous and incompressible, and that the fluid's motion is a steady flow with no rotation.

Two important concepts regarding ideal fluid flow through a pipe of nonuniform size are as follows:

1. The flow rate (volume flux) through the pipe is constant; this is equivalent to stating that the product of the cross-sectional area $A$ and the speed $v$ at any point is a constant. This result is expressed in the **equation of continuity for fluids:**

$$A_1 v_1 = A_2 v_2 = \text{constant} \tag{14.7}$$

2. The sum of the pressure, kinetic energy per unit volume, and gravitational potential energy per unit volume has the same value at all points along a streamline. This result is summarized in **Bernoulli's equation:**

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$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant} \tag{14.9}$$